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On the Gauge and BRST Invariance of the Chiral QED with Faddeevian Anomaly

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Abstract Chiral Schwinger model with the Faddeevian anomaly is considered. It is found that imposing a chiral constraint this model can be expressed in terms of chiral boson. The model when expressed in terms of chiral boson remains anomalous and the Gauss law of which gives anomalous Poisson brackets between itself. In spite of that a systematic BRST quantization is possible. The Wess-Zumino term corresponding to this theory appears automatically during the process of quantization. A gauge invariant reformulation of this model is also constructed. Unlike the former one gauge invariance is done here without any extension of phase space. This gauge invariant version maps onto the vector Schwinger model. The gauge invariant version of the chiral Schwinger model for $a = 2$ has a massive field with identical mass however gauge invariant version obtained here does not map on to that.

Keywords Chiral QED · BRST invariance · Faddeevian anomaly

1 Introduction

Symmetry plays a fundamental role in physics. Some times symmetry of a given theory may be broken and that has a profound consequences. Gauge symmetry of a theory is of particular interest in this context. Absence of gauge symmetry invites anomaly in a theory. There have been considerable efforts in the understanding of anomaly in quantum field theory [1–14]. The studies of chiral Schwinger model and anomalous Schwinger model [11] are worth mentionable in this respect. It is the anomaly that removed the long suffering of chiral Schwinger model from non-unitarity. Credit went to Jackiw and Rajaraman—those

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who offered a consistent analysis of this model in a gauge non-invariant manner [1]. However a gauge invariant version is always favorable because of its increased symmetry. This work is an illustration on gauge as well as BRST invariance of chiral Schwinger mode with Faddeevian [5, 6] type of anomaly.

In terms of constraint [15], a gauge invariant theory is defined as a theory with first class constraint and the presence of second class constraints indicate the breaking of this invariance. The conversion mechanism of second class constraints into a first class was introduced initially by Faddeev and Shatashvili in [6]. The formalism was extended further by Batalin, Fradkin and Vilkovisky [16–20] and became amenable for obtaining BRST invariant effective action. It is known so as BFV formalism. There have been attempts for this conversion in different approaches too. The approaches basically fall into two independent classes. In one class extension of phase space through the introduction of auxiliary fields is required [16–22]. The other class however does not require this extension [23, 24].

Study of free chiral boson [4, 25–28] as well as gauged chiral boson [8–10, 26] are very interesting in connection with the restoration of gauge invariance because of its peculiar constraint structure. To be precise chiral constraint[1] shows nonvanishing Poisson bracket with itself. Gauge invariant reformulation of free chiral boson and gauged chiral boson are considered by several authors in different time [20–24, 29, 30]. It is known that two independent version of gauged chiral boson are available in the literature. The oldest one is the version proposed by Jackiw and Rajaraman [1]. We should mention here that Hagen initially gave the chiral generation of Schwinger model [31]. The model however failed to maintain the unitarity. Jackiw and Rajaraman saved the model introducing anomaly [1] within it and gave a consistent hamiltonian description of that. Mitra suggested an alternative gauge non-invariant version of gauged chiral boson [9, 10]. The anomaly of which corresponds to Faddeevian type [5–7] where gauss law constraint gave nonvanishing Poisson bracket among itself. The model attracted several attention because of this special type of constraint structure.

BRST invariant reformulation of Jackiw-Rajaraman version of chiral Schwinger model is done in [20, 29]. However the gauge (BRST) invariant reformulation is lacking for the chiral Schwinger model where anomaly is Faddeevian like. It would be worthy to have a systematic development where gauge invariance gets restored and the Wess-Zumino term comes out automatically during the process. With this in view and also as a pedagogical illustration of the BVF formalism effort has been made to obtain a BRST invariant effective action of this model. The work will certainly demonstrate the power of BFV formalism once more. This new study would be instrumental for future studies towards unitarity and renormalization of this model. Gauge invariant reformulation of this model also carried out in its usual phase space using Mitra- Rajaraman prescription in order to have a better feeling about the difference between gauge invariance in the usual phase space and the extended phase space. In [32], Shatashvili considered the non-Abelian version of gauge invariant chiral Schwinger model where he observed an special feature in connection with the reduction of interacting degrees of freedom of this model for $a = 2$. That work also showed that the mass term in that was identical to the mass term of the model considered here for that specified value of a. At the first sight one may think that these two models are identical but that is not the case. To get a clear picture we compare our present development with the work of Shatashvili [32].

[1] Chiral constraint is a relation between the momenta and coordinate of the chiral boson. Mathematically it is $T(x) = \pi_\phi - \Phi' \approx 0$. The Poisson bracket between $T(x)$ and Ty is $[T(x), T(y)] = -2\delta'(x - y)$.

The paper is organized as follows. Section 2, contains a brief review of the model in connection with the bosonization of the fermionic version of chiral Schwinger model and imposition of a chiral constraint to express it in terms of chiral boson. In Sect. 3, a brief introduction of the BFV formalism is given and then it is applied to this model to obtain the BRST invariant reformulation of that. Mitra-Rajaraman prescription is used in Sect. 4 to obtain a gauge invariant reformulation of the same model. In Sect. 5, a comparison is made between the result obtain in Sect. 4, and the known gauge invariant version of the usual chiral Schwinger model for $a = 2$. Section 6 contains a brief discussion over the work.

2 Bosonization of Fermionic Model and Imposition of Chiral Constraints

Chiral Schwinger model is described by the following generating functional

$$Z[A] = \int d\psi d\bar{\psi} e^{\int d^2x \mathcal{L}_f}, \tag{1}$$

with

$$\begin{aligned}\mathcal{L}_f &= \bar{\psi}\gamma^\mu[i\partial_\mu + e\sqrt{\pi}A_\mu(1-\gamma_5)]\psi \\ &= \bar{\psi}_R\gamma^\mu i\partial_\mu\psi_R + \bar{\psi}_L\gamma^\mu(i\partial_\mu + 2e\sqrt{\pi}A_\mu)\psi_L.\end{aligned} \tag{2}$$

The right handed fermion remains uncoupled in this type of chiral interaction. So integration over this right handed part leads to field independent counter part which can be absorbed within the normalization. Integration over left handed fermion leads to

$$Z[A] = \exp\left[\frac{ie^2}{2}\int d^2x A_\mu\left[M^{\mu\nu} - (\partial^\mu + \tilde{\partial}^\mu)\frac{1}{\Box}(\partial^\nu + \tilde{\partial}^\nu)\right]A_\nu\right]. \tag{3}$$

$M_{\mu\nu} = ag_{\mu\nu}$, for Jackiw-Rajaraman regularization where the parameter a represents the regularization ambiguity and

$$M_{\mu\nu} = \begin{pmatrix} 1 & -1 \\ -1 & -3 \end{pmatrix}\delta(x-y),$$

for an alternative version proposed in [9, 10]. Writing down the generating functional in terms of the auxiliary field $\phi(x)$ it turns out to the following

$$Z[A] = \int d\phi e^{i\int d^2x \mathcal{L}_B}, \tag{4}$$

with

$$\begin{aligned}\mathcal{L}_B &= \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) + e(g^{\mu\nu} - \epsilon^{\mu\nu)}\partial_\mu\phi A_\nu + \frac{1}{2}e^2A_\mu M^{\mu\nu}A_\nu \\ &= \frac{1}{2}(\dot{\phi}^2 - \phi'^2) + e(\dot{\phi} + \phi')(A_0 - A_1) + \frac{1}{2}e^2(A_0^2 - 2A_0A_1 - 3A_1^2).\end{aligned} \tag{5}$$

Here $\epsilon^{01} = -\epsilon_{01} = 1$ and the Minkowski metric $g^{\mu\nu} = \text{diag}(1, -1)$.

Equation (5) was initially found in [9] where Mitra termed it as chiral Schwinger model with Faddeevian regularization. In [9], we find that the Gauss law constraint of this theory is

$$G = \pi_1' + e(\pi_\phi + \phi'). \tag{6}$$

It is found there that the Poisson bracket between G(x) and G(y) is

$$[G(x), G(y)]) = 2\delta(x-y)'. \tag{7}$$

This Poisson (7) was found to gave the vanishing contribution for the usual chiral Schwinger model [1]. Faddeev initially noticed that anomaly made Poisson bracket between $G(x)$ and $G(y)$ nonzero [5, 6]. The constraint became second class itself and gauge invariance was lost. He, however, argued that it would be possible to quantize the theory but in this situation system may posses more degrees of freedom.

From the standard definition, the momentum corresponding to the field ϕ is found out to be

$$\pi_\phi = \dot\phi + e(A_0 - A_1). \tag{8}$$

The following Legendre transformation

$$H_B = \int d^2x[\pi_\phi\dot\phi - \mathcal{L}_B], \tag{9}$$

leads to the hamiltonian density

$$\begin{aligned}\mathcal{H}_B = {} & \frac{1}{2}[\pi_\phi - e(A_0 - A_1)]^2 + \frac{1}{2}\phi'^2 - 2e\phi'(A_0 - A_1) \\ & - \frac{1}{2}e^2(A_0^2 - 2A_0A_1 - 3A_1^2).\end{aligned} \tag{10}$$

In order to suppress one chirality at this stage we impose the chiral constraint

$$\omega(x) = \pi_\phi(x) - \phi'(x) \approx 0. \tag{11}$$

It is a second class constraint itself since

$$[\omega(x), \omega(y)] = -2\delta'(x-y). \tag{12}$$

After imposing the constraint $\omega(x) \approx 0$, into the generating functional we arrived at the following

$$\begin{aligned}Z_{CH} &= \int d\phi d\pi_\phi \delta(\pi_\phi - \phi')\sqrt{\det[\omega, \omega]}e^{i\int d^2x(\pi_\phi\dot\phi - \mathcal{H}_B)} \\ &= \int d\phi e^{i\int d^2x \mathcal{L}_{CH}},\end{aligned} \tag{13}$$

with

$$\mathcal{L}_{CH} = \dot\phi\phi' - \phi'^2 + 2e(A_0 - A_1)\phi' + 2e^2A_1^2. \tag{14}$$

We obtained the gauged lagrangian for chiral boson from the bosonized lagrangian with Faddeevian regularization [9] just by imposing the chiral constraint in its phase space. Harada in

[8], obtained the same type of result for the usual chiral Schwinger model with one parameter class of regularization proposed by Jackiw and Rajaraman [1]. The lagrangian (14) can be thought of as the gauged version of chiral boson described by Floreanini and Jackiw [4]. The constraint analysis and the phase space structure corresponding to this model is available in [10]. In Ref. [10], we found that the theory (14) describes a massive boson through the equation

$$[\Box + 4e^2]A_1 = 0 \tag{15}$$

with square of the mass $m^2 = 4e^2$. Equation (15) was interpreted there as the photon acquired mass and the fermion got confined.

3 BRST Invariant Reformulation Using BFV Formalism

Before we apply BFV formalism on this model it would be useful for the reader to give a brief introduction of the formalism. BRST invariance essentially means to enlarge the Hilbert space of a gauge theory in order to restore the symmetry of a gauge fixed action in that enlarged space. It is very effective when one tries to study the renormalization property of a theory. One generally exploit the BRST symmetry instead of exploiting the original gauge symmetry. The discovery of this symmetry raised the ghost field to a prominent position. It mixes the ghost with the other fields of the theory and therefore all the fields including the ghosts can be regarded as a different components of a single geometrical object.

The combined formalism of Batalin, Fradkin and Vilkovisky [16–20] for quantization of a system is based on the idea that a system with second class constraint can be made effectively first class in the extended phase space which finally helps to find BRST invariant effective action. The field needed for this conversion ultimately turns out into the Wess-Zumino scalar with the proper choice of gauge condition, as pointed out by Fugiwara, Igarashi and Kubo [18]. What follows next is a brief description of the general BFV formalism for obtaining a BRST invariant action.

Let us consider a canonical hamiltonian described by the canonical pairs (p_i, q^i), $i = 1, 2, \ldots, N$. The pairs are subjected to a set of constraints $\Omega_a \approx 0$, $a = 1, 2, \ldots, n$, and it is assumed that the constraints satisfy the following algebra [20, 21].

$$[\Omega_a, \Omega_b] = i\Omega_c U^c_{ab}, \tag{16}$$

$$[H_c, \Omega_a] = i\Omega_b V^b_c, \tag{17}$$

then n no of additional condition $\Phi_a \approx 0$ with $det[\Phi_a, \Omega_b] \neq 0$ have to be imposed in order to single out the physical degrees of freedom. The constraints $\Omega_a \approx 0$ and $\Phi_a \approx 0$, together with hamiltonian equation of motion is obtained from the action

$$S = \int dt[p_i\dot{q}^i - H_c(p_i, q^i) - \lambda^a\Omega_a + \pi_a\Phi^a], \tag{18}$$

where λ^a and π_a are Lagrange multiplier fields and these two satisfy the relation $[\lambda^a, \pi_b] = i\delta^a_b$.

Now introducing one pair of canonical ghost field $(C^a, \bar{P}_a)$ and one pair of canonical anti-ghost field $(P^a, \bar{C}_a)$ for each pair of constraints an equivalence can be made to the initial theory with constraints in the reduced phase space. So the quantum theory can be described by the partition function where the action [16–22] in its numerator will be

$$S_{qf} = \int dt[p_i\dot{q}^i + \pi^a\dot{\lambda}_a + \bar{P}^a\dot{C}_a + \bar{C}^a\dot{P}_a - H_{BRST} + i[Q,\psi]]. \tag{19}$$

H_{BRST} is the minimal hamiltonian [16, 17]as termed by Batalin and Fradkin, is defined by

$$H_{BRST} = H_c + \bar{P}_a V_b^a C^b. \tag{20}$$

The BRST charge Q and the fermionic gauge fixing function ψ are respectively given by [20–22]

$$Q = C^a\omega_a - \frac{1}{2}C^b C_c U_{ab}^c \bar{P}^a + P^a\pi_a, \tag{21}$$

$$\psi = \bar{C}_c\chi^a + \bar{P}^a\lambda^a, \tag{22}$$

where χ_a's are expressed through the gauge fixing condition

$$\Phi_a = \dot{\lambda}_a + \chi_a. \tag{23}$$

Let us now concentrate on the BRST invariant reformulation of the lagrangian (14). In order to do that we need to know the constraint structure of the theory. The details of which is available in [9, 10]. Here we are giving the relevant portion as required for our purpose. In [10], we find that the momenta corresponding to the fields A_0, A_1 and ϕ are.

$$\pi_\phi = \phi', \tag{24}$$

$$\pi_1 = \dot{A}_1 - A_0', \tag{25}$$

$$\pi_0 = 0. \tag{26}$$

It is known that $\pi_0 = 0$ and $\pi_\phi = \phi'$ are the primary constraints of the theory.

The effective hamiltonian follows from the equations of motion is

$$H_P = \int dx[\mathcal{H}_C + u\pi_0 + v(\pi\phi - \phi')], \tag{27}$$

where

$$\mathcal{H}_C = \frac{1}{2}\pi_1^2 + \pi_1 A_0' + \phi^2 - 2e(A_0 - A_1)\phi' - 2e^2A_1^2 \tag{28}$$

Here u and v are two required lagrange multipliers. The preservation of the constraints leads to two other constraints

$$G = \pi_1' + 2e\phi' \approx 0, \tag{29}$$

$$-2e^2(A_1 + A_0)' \approx 0. \tag{30}$$

The multipliers u and v are found out to be

$$u = -(\pi_1 + A_0'), \tag{31}$$

$$v = \phi - e(A_0 - A_1). \tag{32}$$

Therefore, the theory under consideration contains four constraints in its phase space. Precisely, the constraints are

$$\omega_1 = \pi_\phi - \phi' \approx 0, \tag{33}$$

$$\omega_2 = \pi_0 \approx 0, \tag{34}$$

$$\omega_3 = \pi_1' + 2e\phi' \approx 0, \tag{35}$$

$$\omega_4 = -2e^2(A_1 + A_0)' \approx 0. \tag{36}$$

These four constraints form a second class set and the closures of the constrains with respect to the hamiltonian (27) are given by

$$\dot{\omega}_1 = \omega_1', \tag{37}$$

$$\dot{\omega}_2 = \omega_3 - \omega_2' + e\omega_1, \tag{38}$$

$$\dot{\omega}_3 = \omega_4 - e\omega_1', \tag{39}$$

$$\dot{\omega}_4 = 2e^2\omega_2'. \tag{40}$$

To obtain a BRST invariant reformulation we need to convert the second class set of constraints into a first class set. With this in view, we introduce four auxiliary fields ψ , η, π_ψ and π_η and fields are such that they satisfy the following canonical condition

$$[\eta(x), \pi_\eta(y)] = \delta(x-y), \tag{41}$$

$$[\psi(x), \pi_\psi(y)] = \delta(x-y). \tag{42}$$

The fields used here are known as Batalin-Fradkin (BF) fields. The constraints (33), (34), (35) and (36), with some suitable linear combination of the BF fields get converted into first class set as follows

$$\tilde{\omega}_1 = \pi_\phi - \phi' + \pi_\psi + \psi', \tag{43}$$

$$\tilde{\omega}_2 = \pi_0 - \pi_\eta, \tag{44}$$

$$\tilde{\omega}_3 = -2e\psi' + 2e\phi' + \pi_1' - \pi_\eta', \tag{45}$$

$$\tilde{\omega}_4 = -2e^2(A_0 + A_1)' - 2e^2\eta'. \tag{46}$$

The above four first class constraints will be found consistent with the first class hamiltonian if these new first class set satisfy the same closures as their ancestor did with the hamiltonian (27). Precisely, the conditions are

$$\dot{\tilde{\omega}}_1 = \tilde{\omega}_1', \tag{47}$$

$$\dot{\tilde{\omega}}_2 = \tilde{\omega}_3 - \tilde{\omega}_2' + e\tilde{\omega}_1, \tag{48}$$

$$\dot{\tilde{\omega}}_3 = \tilde{\omega}_4 - e\tilde{\omega}_1', \tag{49}$$

$$\dot{\tilde{\omega}}_4 = 2e^2\tilde{\omega}_2'. \tag{50}$$

First class hamiltonian is obtained by the appropriate insertion of the BF fields within the hamiltonian (27) and it is given by $\tilde{H}=H_P+H_{BF}$. Here H_{BF} is a polynomial of ψ , η, π_ψ

and π_η that extend the phase space respecting the closures (47), (48), (49) and (50). We find that H_{BF} for this system will be

$$H_{BF} = \int dx[-2e\eta\psi' + e(\pi_\psi + \psi')\eta + \frac{1}{2}(\pi_\eta^2 + \pi_\psi^2 + \psi'^2)]. \tag{51}$$

We now introduce four pairs of ghost $(C_i, \bar{P}^i)$ and four pairs of anti-ghost $(P_i, \bar{C}^i)$ fields. Four pairs of multiplier fields (N^i, B_i) are also needed. The pairs satisfy the following canonical relations

$$[C_i, \bar{P}^j] = [P^i, \bar{C}_j] = [N^i, B_j] = i\delta^i_j\delta(x-y), \quad i = 1, 2, 3, 4. \tag{52}$$

From the definition we can write BRST invariant hamiltonian

$$H_U = H_{BRST} - i[Q, \psi], \tag{53}$$

where H_U is the unitarizing hamiltonian, Q is the BRST charge and ψ's are the gauge fixing functions. Note that the BRST charge Q is a nilpotent operator and it satisfies the equation

$$Q^2 = [Q, Q] = 0. \tag{54}$$

The definition of Q in this formalism is

$$Q = \int (B_i P^i + C_i\tilde{\omega}^i)dx, \tag{55}$$

and the definition of gauge fixing function ψ is

$$\psi = \int (\bar{C}_i X^i + P_i N^i)dx. \tag{56}$$

The BRST invariant hamiltonian for the theory with which we are dealing with is

$$\begin{aligned} H_{BRST} = H_P + H_{BF} + \int dx(&-\bar{P}_1 C_1' + \bar{P}_3 C_2 + \bar{P}_2 C_2' + e\bar{P}_1 C_2 + \bar{P}_4 C_3 \\ &- e\bar{P}_1' C_3 + 2e^2\bar{P}_2' C_4). \end{aligned} \tag{57}$$

It would be helpful to write down the generating functional that ultimately leads to an effective action with the elimination of some fields by Gaussian integration. The generating functional reads

$$Z = \int [D\mu]e^{iS}. \tag{58}$$

Here the expression of S is

$$S = \int d^2x[\pi_\phi\dot{\phi} + \pi_1\dot{A}_1 + \pi_0\dot{A}_0 + \pi_\psi\dot{\psi} + \pi_\eta\dot{\eta} + \bar{P}_i\dot{C}^i + \bar{C}_i\dot{P}^i + B_i\dot{N}^i - H_U], \tag{59}$$

where $[D\mu]$ is the Liouville measure in the extended phase space. We are now in a position to fix up the gauge conditions

$$\chi_1 = \pi_\phi - \phi', \tag{60}$$

$$\chi_2 = -\dot{N}^2 + A_0, \tag{61}$$

$$\chi_3 = \frac{B_3}{2} - A_1', \tag{62}$$

$$\chi_4 = \pi_\eta - \dot{N}^4. \tag{63}$$

When we substitute the simplified form of H_{final} obtained after plugging the gauge fixing conditions (60), (61), (62) and (63) in the action (59), we get the explicit expression of S:

$$\begin{aligned} S = \int d^2x \Bigg[& \pi_\phi \dot{\phi} + \pi_\psi \dot{\psi} + \pi_\eta \dot{\eta} + \pi_1 \dot{A}_1 + \pi_0 \dot{A}_0 + \bar{P}_i \dot{C}^i + \bar{C}_i \dot{P}^i + B_i \dot{N}^i \\ & - \bigg(\frac{\pi^2}{2} + \pi A_0' + e\phi'(A_0 - A_1) + 2e^2 A_1^2 - \pi_0(\pi_1 + A_0') + \pi_\phi \phi' \\ & - e\pi_\phi(A_0 - A_1) - e\eta\psi' + e\pi_\psi \eta + \frac{1}{2}(\pi_\eta^2 + \pi_\psi^2 + \psi'^2) + B_i \chi^i + \tilde{\omega}_i N^i \\ & - \bar{P}_i P^i - \bar{P}_1 C_1' + \bar{P}_3 C_2 + \bar{P}_2 C_2' + e\bar{P}_1 C_2 + \bar{P}_4 C_3 - e\bar{P}_1' C_3 + 2e^2 \bar{P}_2' C_4 \\ & - C_3 \bar{C}'' - \bar{C}_2 \dot{P}^2 - \bar{C}_4 \dot{P}^4 + 2e^2 C^4 \bar{C}_4 - 2C'\bar{C}_1 - e^2 C^2 \bar{C}_2 \bigg) \Bigg]. \end{aligned} \tag{64}$$

Here i runs from 1 to 4. Our next task is to simplify (58) through the elimination of some fields and that will lead us to our desired result. A careful look reveals that here exists a simplification

$$\int d^2x (B_i N^i + \bar{C}_i \dot{P}^i) = -i \left[Q, \int d^2x (\bar{C}_i \dot{N}^i) \right] \tag{65}$$

with be Legendre transformation $B^i \to B^i + \dot{N}^i$. However the simplification corresponding to $i = 1$ suffices in this situation. More simplification follows from the elimination of the fields $\pi_0, \pi_1, \pi_\eta, B_1, B_2, B_4, A_0, N^1, N^2, N^4, P_1, \bar{P}^1, P_2, \bar{P}^2, P_4, \bar{P}^4, P_1, \bar{P}^1, C_1, \bar{C}^1, C_2$ and $\bar{C}^2$ by Gaussian integration. Ultimately we reach to a very simplified form of the generating functional (58) that contains the following effective action in its numerator.

$$\begin{aligned} S_{eff} = \int d^2x (& \dot{\phi}\phi' - \phi'^2 + 2e^2 A_1^2 - \psi'^2 - \dot{\psi}\psi' + \frac{1}{2}(\dot{A}_1 - A_0')^2 \\ & + 2e\phi'(A_0 - A_1) - 2e\psi'(A_1 + A_0) + \partial_\mu B A^\mu + \frac{1}{2}\alpha B^2 \\ & + \partial_\mu \bar{C} \partial^\mu C. \end{aligned} \tag{66}$$

We have used few redefinition of fields, e.g., $N_3 = A_0$ and P^3=$\dot{C}_3$ to reach to the result (66). Since after elimination there is no other B's and C's except B_3 and C_3 we are free to read them as B and C. It is now time to check the invariance of the action (66). A little algebra shows that the action is invariant under the transformation

$$\begin{aligned} &\delta A_1 = -\lambda C', \qquad \delta A_0 = \delta N_3 = \lambda \dot{C}, \\ &\delta\psi = \lambda C, \qquad \delta\bar{C} = \lambda B, \qquad \delta C = 0. \end{aligned} \tag{67}$$

It is to be mentioned that the fields satisfy the following Euler-Lagrange equation

$$\partial_-\phi - \partial_+\psi - 2eA_1 = 0. \tag{68}$$

We can identify easily the Wess-Zumino term for this theory which is

$$\mathcal{L}_{wz} = -\dot{\psi}\psi' - \psi'^2 - 2e\psi'(A_0 + A_1). \tag{69}$$

It is interesting to see this automatic appearance of this Wess-Zumino term during the process of obtaining the BRST invariant action. One point we should mention here that the choice of gauge condition is very crucial. One may miss to get Wess-Zumino term otherwise.

4 Gauge Invariant Reformulation without Extending the Phase Space

The formalism of making a theory gauge invariant by the reduction of the number of second class constraint was first developed by Mitra and Rajaraman [23, 24]. The formalism strictly depends on the constraint structure of the theory. Depending on the constraint structure of the theory different gauge invariant version is possible for a particular theory. No extension of phase space is needed in this formalism. So the physical contents of all the gauge invariant actions remains the same. In [23, 24], the authors gave a reasonably general theory relating to a large class of systems with second class constraints to corresponding class of gauge invariant systems having the same dynamical content. A gauge theory in a generalized sense means a theory with some first class constraints. To covert it into an equivalent second class system is well known. One generally fix the gauge, i.e., impose a suitable number of gauge fixing conditions. These gauge fixing conditions together with the original first class set of constraint form a second class set and the theory gets converted into an equivalent second class system. An inverse procedure is suggested in [23, 24] where a formalism is developed for construction of a gauge invariant system equivalent to a given second class theory. The authors argued there as follows. If a dynamical system possess $2n$ constraints and the constraints all together form a second class set and if n of these constraints are found to have mutually vanishing Poisson brackets then these n constraints can be used as gauge generator of the gauge invariant reformulation. The remaining n constraints may be thought of as the gauge fixing condition. The hamiltonian needs the required modification accordingly. So in [23, 24] the authors suggested to reduce half of the constraint from a second class set of constraint retaining the first class set only in order to get the gauge invariant reformulation. The obtained gauge invariant theory can be treated in the similar way as any standard gauge invariant theory is treated. What follows next is the application of the formalism in the presently considered mode.

To apply this formalism in a model it is essential to know the constraint structure of that theory. In our case which is already given in Sect. 3. We have seen there that the phase space of the model described by the lagrangian (14) contains four constraints. In Sect. 3, those constraints are given in (33), (34), (35) and (36). Note that the combination $\omega_2 \approx 0$ and $\omega_3 \approx 0$ form a first class set. If we retain only these two constraints as stated above, following the suggestion available in [23, 24], we require a modification of the hamiltonian density of the second class system (27) in the following manner in order to get a first class system.

$$\begin{aligned}\mathcal{H} = {}& \frac{1}{2}\pi_1^2 + \pi_1 A_0' - e(A_0 - A_1)\phi' + 2e^2 A_1^2 + \pi_\phi\phi' - e\pi_\phi(A_0 - A_1) \\ & + e(\pi_\phi - \phi')(A_0 + A_1) + \frac{1}{2}(\pi_\phi - \phi')^2 + u\pi_0.\end{aligned} \tag{70}$$

The modification certainly keeps the physical contents of the theory intact. This modified hamiltonian density (70) contains only the two first class constraints $\omega_2 \approx 0$ and $\omega_3 \approx 0$. The equation of motion with respect to the hamiltonian (70) are found out as follows

$$\dot{\phi} = [\phi, \mathcal{H}] = \pi_\phi + 2eA_1, \tag{71}$$

$$\dot{A}_0 = [A_0, \mathcal{H}] = -u, \tag{72}$$

$$\dot{A}_1 = [A_1, \mathcal{H}] = \pi_1. \tag{73}$$

A straightforward calculation leads to the lagrangian density corresponding to the first class theory with which we are interested in.

$$\mathcal{L}_1 = \pi_\phi\dot{\phi} + \pi_1\dot{A}_1 + \pi_0\dot{A}_0 - \left[\frac{\pi_1^2}{2} + \pi_1 A_0' + 2eA_1\pi_\phi + \pi_\phi\phi' - 2eA_0\phi' \right.$$
$$\left. + \frac{1}{2}(\pi_\phi - \phi')^2 + u\pi_0 + 2e^2A_1^2\right]. \tag{74}$$

After a little algebra the lagrangian density acquires a very simplified form

$$\mathcal{L}_2 = \frac{1}{2}(\dot{\phi}^2 - \phi'^2) - 2e(A_1\dot{\phi} - A_0\phi') + \frac{1}{2}(\dot{A}_1 - A_0')^2. \tag{75}$$

The lagrangian density (75), is consistent with the hamiltonian density (70), and the equations of motion (71), (72) and (73). To see whether the lagrangian density (75) stems out from the modified hamiltonian density (70) contains only the two first class (34) and (35) in its phase space let us calculate the momenta corresponding to the field A_0

$$\pi_0 = \frac{\partial \mathcal{L}_2}{\partial \dot{A}_0} = 0. \tag{76}$$

It gives back the primary constraint (35) and the preservation of this once again gives the Gauss law constraint

$$G = \pi_1' + 2e\phi' \approx 0. \tag{77}$$

No other constraints come out from the preservation of (77). These two first class constraints help us to construct the gauge transformation generator. The generator is given by

$$\mathcal{G} = \int dx(\lambda_1\omega_1 + \lambda_2\omega_2). \tag{78}$$

Here λ_1 and λ_2 are two arbitrary parameters. The transformations evolved out of the generator (78) for the fields ϕ, A_1 and A_0 respectively are

$$\delta\phi = 0, \qquad \delta A_1 = -\lambda_1', \qquad \delta A_0 = -\lambda_2. \tag{79}$$

A little algebra shows that under the transformation (79), the lagrangian (75) remains invariant provided the parameter satisfy the relation

$$\lambda_2 = \dot{\lambda}_1. \tag{80}$$

A note worthy thing is that this transformation is equivalent to the transformation $A_\mu \to A_\mu + \frac{1}{2e}\partial_\mu\lambda$. There is some thing interesting that we must mention here. The first class lagrangian that comes out from our investigation is the bosonized lagrangian of the well known vector Schwinger model [33, 34]. Here coupling strength is $2e$. It does not come as a great surprise because the theoretical spectrum of the model under consideration is identical to the vector Schwinger model. To be precise, both the models contain the massive boson with mass $m = 2e$.

We have mentioned earlier that the gauge invariant reformulation follows from this prescription depends crucially on the constraint structure of the model. There are other possibilities to get first class set of constraints from the set of constraints (33), (34), (35) and (36). However that possibilities fail to give consistent first class theories.

5 Comparison of the Result Obtained in Sect. 4 with the Gauge Invariant Chiral Schwinger Model for $a = 2$

Let us compare our result with the work of the Shatashvili [32] because seeing their apparent similarities at a first glance one may think that these two results are identical. But a careful look revels that this is not so. In his work Shatashvili considered the non-Abelian gauge invariant version of the chiral Schwinger model and showed that the interacting degrees of freedom gets reduced if the choice $a = 2$ is made. For $a = 2$, the mass term of Shatashvili's model become identical to our model but there lies a basic difference which we would like to address. Here we consider the gauge invariant Abelian bosonized version of that model [35] because this version would be compatible for comparison with our work. Unlike the non-Abelian version the Abelian version it is exactly solvable too.

It is described by the lagrangian density

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) + e(g^{\mu\nu} - \epsilon^{\mu\nu)}\partial_\mu\phi A_\nu + \frac{1}{2}ae^2 A_\mu A^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + L_{WESS}, \tag{81}$$

where L_{WESS} is given by

$$\mathcal{L}_{WESS} = \frac{1}{2}(a-1)(\partial_\mu\eta)(\partial^\mu\eta) + e[(a-1)g^{\mu\nu} + \epsilon^{\mu\nu}]\partial_\mu\eta A_\nu. \tag{82}$$

The lagrangian is invariant under the gauge transformation $A_\mu \to A_\mu + \frac{1}{e}\partial_\mu\Lambda$, $\phi \to \phi + \Lambda$, $\eta \to \eta - \Lambda$. The momenta corresponding to the fields A_0, A_1 and ϕ and η are

$$\pi_\phi = \dot{\phi} + e(A_0 - A_1), \tag{83}$$

$$\pi_1 = \dot{A}_1 - A_0', \tag{84}$$

$$\pi_0 = 0, \tag{85}$$

$$\pi_\eta = (a-1)\dot{\eta} + e[(a-1)A_0 + A_1]. \tag{86}$$

Equation (83), (84) and (85) are independent of the parameter a. The choice $a = 2$ brings change only in (86) and with that choice that turns into

$$\pi_\eta = \dot{\eta} + e(A_0 + A_1). \tag{87}$$

A straightforward calculation shows that the canonical Hamiltonian density for the model with $a = 2$ is

$$\mathcal{H}_c = \frac{1}{2}[\pi_1^2 + \pi_\phi^2 + \phi'^2] - eA_1(\pi_\phi - \phi') + 2e^2A_1^2 + \frac{1}{2}[\pi_\eta^2 + \eta^2] - eA_1(\pi_\eta + \eta')$$
$$- A_0(\pi_1' + e[(\pi_\phi - \phi') - (\pi_\eta + \eta')]. \tag{88}$$

The phase space of the model contains the following two constraints [35]

$$\Omega_1 = \pi_0 \approx 0, \tag{89}$$
$$\Omega_2 = \pi_1' + e(\pi_\phi - \phi') - e(\pi_\eta + \eta') \approx 0. \tag{90}$$

The constraint (90) appears as a secondary constraint in order to preserve the constraint (89). The two constraints are first class. The first class constraints shows a clear indication of reduction of degrees of freedom because to quantize the theory two gauge fixing conditions are to be needed. Bosonized version of Vector Schwinger model (75), appeared out as the gauge invariant version of chiral Schwinger model with Faddeevian anomaly in Sect. 4, contains the following two constraint

$$\omega_{VS1} = \pi_0 \approx 0, \tag{91}$$
$$\omega_{VS2} = \pi_1' + 2e\phi' \approx 0. \tag{92}$$

The Hamiltonian density of this bosonized version of vector Schwinger model (75) comes out to be

$$\mathcal{H}_{VS} = \frac{1}{2}(\pi_1^2 + \pi_\phi^2 + \phi'^2) + \pi_1 A_0' + 2e(A_1\pi_\phi - A_0\phi'). \tag{93}$$

It is true that both the models are gauge invariant and the massive fields which comes out form (93) and (88) looks almost identical. Square of the mass of the boson in each case is $m^2 = 4e^2$. However the Hamiltonian (88) cannot be made free from Wess-Zumino field η using the constraints (89) and (90) and the constraints (89) and (90) also do not map on to the constraints of the vector Schwinger model. On the contrary the Gauge invariant version as obtained in (75), using Mitra-Rajaraman prescription, does not contain this type of field. Here gauge invariance is resulted in the usual phase space.

6 Discussion

Gauge invariant reformulation of chiral Schwinger with Faddeevian anomaly has been carried out in two different directions. In the first case BFV prescription [16–20] is followed which needs an extension of phase space. The process certainly keeps the physical contents of the theory intact. The fields needed for the extension keep themselves allocated in the unphysical sector of the theory. In this prescription we not only get a BRST invariant effective action but also appropriate Wess-Zumino term appears automatically during the process. In the second approach Mitra-Rajaraman prescription [23, 24] is followed to obtain a gauge invariant action. In this situation we have to be restricted on the gauge invariance only because the formalism developed till now is not adequate to obtain BRST invariant action. In spite of the existence of more than one possibilities only a particular possibility leads to a gauge invariant action there. Surprisingly, the other possibilities fail to do so. Only that possibility has explored to obtain gauge invariant reformulation which renders a very interesting result. The gauge invariant model that comes out is the lagrangian of well known vector Schwinger model [33, 34] and gauge invariance of which is obvious. It is conclusively shown here too.

It is true that the gauge non-invariant version of this model under consideration too contains a massive boson like vector Schwinger model [33, 34]. We have already mentioned it. But the explicit mapping of this model onto the vector Schwinger model is a new and novel result. The counting of degrees of freedom also found to be consistent. It would be interesting to investigate how a particular Faddeevian regularized version of the chiral Schwinger model maps onto the vector Schwinger model in its gauge invariant version. We compare the gauge invariant lagrangian obtained in Sect. 4 with the gauge invariant version of the Abelian chiral Schwinger model setting $a = 2$ in (75). Both the model is gauge invariant and contains a massive field with the same mass. But for the former one gauge invariance has occurred in its usual phase space whereas for the later it does occur in the extended phase space.

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